Institut für Produktion und
Industrielles Informationsmanagement

Universität Duisburg-Essen (Campus: Essen)
Fachbereich 5: Wirtschaftswissenschaften
Universitätsstraße 9, 45141 Essen
Tel.: ++49 (0) 201 / 183 - 4007
Fax: ++49 (0) 201 / 183 - 4017

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Moral Hazard in JIT Production Settings

– A Reconstruction from the Structuralist Point of View –

Dipl.-Kfm. Adem Alparslan

Univ.-Prof. Dr. Stephan Zelewski

E-Mail: {Adem.Alparslan|Stephan.Zelewski}@pim.uni-essen.de

Internet: http://www.pim.uni-essen.de/mitarbeiter

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Abstract:

The agency theory is concerned with problems resulting from conflicts of interest that emerge in contractual relationships when the involved parties are differently informed or uncertain. The objective of agency models is either to explain or to suggest *how* contracts were really designed or should be rationally designed, respectively, in order to deal with precontractual (*adverse selection*) and/or postcontractual problems (*moral hazard*) and *which* behaviour these contracts induce. There are numerous agency models that are directed to analyze production management issues. This paper examines an agency model from ALLES, DATAR and LAMBERT (ADL), which deals with moral hazard and management control problems in Just-in-Time production settings. ADL explain with their model, among other issues, why and how Just-in-Time production systems lead to improvements in worker's productivity. The ADL-model is characterized by a *structuring defect*. This structuring defect results from the formulation of this model according to the conventional conception of theories. At best two components of the ADL-model can be identified: axioms and theorems. Beside this minimal structure no further structure can be identified. This becomes a problem when targeting to answer questions regarding the *nomological essence* of this model. In order to overcome the structuring defect, a theory conception is required which allows the modelling of the essential theory-components. Therefore, the ADL-model is *reconstructed* from the *structuralist point of view*. In this paper, it will be shown, that the reconstruction of this model not only clarifies its nomological essence but also leads to new insights regarding production management models.

Content

1. Introduction

The agency theory[1] has been developed over the last 30 years, with the impacts of this theory being significant. It has been viewed as the neoclassical response to the behaviour of an organization of self-interested employees with conflicting goals in settings with incomplete or uncertain information. It is concerned with problems resulting from conflicts of interest that emerge in contractual relationships when the involved parties are differently informed or uncertain. The objective of the agency theory is either to explain or to suggest *how* contracts were really designed or should be rationally designed, respectively, in order to deal with *precontractual* (adverse selection) and/or *postcontractual* problems (moral hazard) and which behaviour these contracts induce.

Beginning from owner-employee-relationships the agency theory has been applied to economics, business management and business informatics. In recent years, researchers from production and related fields have employed agency theory to a variety of production management issues[2]. For example CRÉMER develops an agency model to analyse the direct incentives provided by Just-in-Time (JIT) principles[3]. PORTEUS and WHANG show with an agency model, by which mechanisms an owner of a firm can reduce or eliminate the negative effects of incongruent goals between a manufacturing and a marketing manager[4].

This paper examines an agency model from ALLES, DATAR and LAMBERT[5] (ADL). ADL explain with their models, among other issues, why and how JIT production systems "*lead to higher worker productivity and efficiency, and process redesign and improvement*"[6]. ADL state ten propositions about JIT production systems, which are de-

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1) Notable contributors to the agency theory include SPENCE (1971); ROSS (1973); HARRIS (1979); HOLMSTRÖM (1979); HOLMSTRÖM (1987). A survey of the agency theory can be found in LAMBERT (2001) and SALANIÉ (1999), pp. 143.

2) A survey of agency models directing to analyze product management issues gives FANDEL (2001).

3) Cf. CRÉMER (1995).

4) Cf. PORTEUS (1991).

5) Cf. ALLES (1995).

6) ALLES (1995), p. 197.

rived by three models. The first model (deterministic production) is not a proper agency model, because there is no information asymmetry between the owner and the worker. The solutions derived by this model serve as a benchmark for the other models. The second and third models are proper agency models. In the second model the owner has only one and in the third model two performance measures for contracting. This paper deals with the second model with one performance measure for contracting.

The ADL-model, which will be denoted in the following as "*mini-theory*", is characterized by a *structuring defect*. This structuring defect results from the formulation of this model according to the conventional conception[7)] of theories. At best two components of the ADL-models can be identified: *axioms* and *theorems*. Beside this minimal structure no further structure can be identified. This becomes a problem when targeting to answer questions regarding the *nomological essence* of these models. In order to overcome the structuring defect, a theory conception is required which allows the modelling of the essential theory-components. Therefore, an ADL-model is reconstructed from the *structuralist point of view* exemplary. Even though all ADL-models are characterized by a structuring defect, only the second model with only one performance measure for contracting will be reconstructed. In the following this model will be denoted as "*mini-theory*". The reconstruction of the third model is left out in this paper, because this wouldn't lead to further insights.

We first give a summary of the mini-theory and show the structuring defect (*Section 2*). Next, the structuralist conception of theories is presented (*Section 3*). Though space limitations preclude an exhaustive description of the structuralist conception of theories, we focus on that issues which are relevant for the argument. Afterwards the structuring defect of the mini-theory is overcome by reconstructing it from the structuralist point of view (*Section 4*). Finally, we discuss the results and show further directions.

In our choice to reconstruct the mini-theory we were guided by *two* goals. *First*, by the reconstruction of this mini-theory we want to analyze the internal structure of a production management model. Therefore, the mini-theory is chosen arbitrarily. The objective

7) The conventional conception of theories is also denoted "*received-view*" or "*statement-view*".

of the reconstruction could also be a theory from other authors. *Second*, we want to demonstrate the usefulness and applicability of the structuralist conception of theories in providing reconstructions of scientific theories from social sciences. It should be emphasized, that we do *not* build a *new* theory but identify a defect of a present theory and show how this defect can be overcome by the structuralist conception of theories.

2 Moral Hazard, JIT Production and the Structuring Defect

2.1 The Mini-Theory

ADL explore in their mini-theory the motivational aspects of JIT (inventory) production systems[8)]. They utilize an agency model, where a risk-neutral principal (owner) contracts with a risk- and effort-averse agent (worker) to supply effort at a single working station. The worker's goal is imperfect aligned with those of the owner (*goal incongruence*) and the worker's effort cannot be observed accurately by the owner (*information asymmetry*).

Within the relationship, the worker must choose an *eff*ort level:

(1) $\text{eff} \in \text{EFF} \subseteq \Re_{\geq 0}$.

However, the owner has to choose the *in*ventory *l*evels:

(2) $\text{inl} \in \text{INL} \subseteq \Re_{\geq 0}$.

The output function *out* is linear additive in the effort level *eff*, inventory level *inl* and a *ran*dom term *ran*:

(3) $\text{out}(\text{eff}, \text{inl}, \text{ran}) = \theta(\text{eff}, \text{inl}) + \text{ran}$

with $\text{ran} \in \text{RAN} \subseteq \Re$.

8) Cf. for the mini-theory ALLES (1995), pp. 179-187.

The random term is normally distributed with mean $\mathrm{E(ran)} = 0$ and variance $\mathrm{Var(ran)} = \sigma^2_{\mathrm{ran}}$.

Consequently, the output is normally distributed for every eff and inl. The output's mean and variance are:

(4) $\mathrm{E}\left(\mathrm{out(eff, inl, ran)}\right) = \theta\left(\mathrm{eff, inl}\right),\ \mathrm{Var(out(eff, inl, ran))} = \sigma^2_{\mathrm{out}}$.

The worker can increase the expected output by choosing a higher effort level, but at diminishing marginal rates[9)]:

(5) $\theta_{\mathrm{eff}}(\mathrm{eff, inl}) > 0$ and $\theta_{\mathrm{eff.eff}}(\mathrm{eff, inl}) < 0$.

Also the increase of the inventory level increases the expected output with diminishing marginal rates:

(6) $\theta_{\mathrm{inl}}(\mathrm{eff, inl}) > 0$ and $\theta_{\mathrm{inl.inl}}(\mathrm{eff, inl}) < 0$.

Following the results from HOLMSTRÖM and MILGROM[10)], ADL assume that the optimal *com*pensation contract will be a *linear function* of the firm's output. That means, the worker will get a fixed payment (*fix*) and a share (*sha*) of the output out(eff,inl,ran):

(7) $\mathrm{com_{sha.fix}(out(eff, inl, ran)) = sha \cdot out(eff, inl, ran) - fix}$

with $\mathrm{sha} \in \Re_{\geq 0}$ and $\mathrm{fix} \in \Re_{\geq 0}$.

The owner is assumed as *risk-neutral* with the NEUMANN-MORGENSTERN utility function u_o:

(8) $$\mathrm{u_o\left(out(eff, inl, ran), com_{sha.fix}(out(eff, inl, ran)), co_{inv}(inl)\right)}$$
$$\mathrm{= out(eff, inl, ran) - com_{sha.fix}(out(eff, inl, ran)) - co_{inv}(inl)}$$

9) $\theta_{\mathrm{eff}}(\mathrm{eff, inl})$ stands for the partial derivate of $\theta(\mathrm{eff, inl})$ with respect to eff.

where out(eff,inl,ran) is the firm's output (whereby it is assumed that the price for an output is equal one), $com_{sha.fix}(out(eff,inl,sta))$ is the worker's compensation and co_{inv}(inl) is the *co*st for the *inv*entory.

The worker has a utility function u_w of CARA type in the exponential (exp) form, which is multiplicatively separable in utility from compensation and disutility from effort:

(9) $$u_w\left(com_{sha.fix}(out(eff,inl,ran)),dis(eff)\right)$$

$$= -\exp(-a(sha \cdot out(eff,inl,ran) - fix - dis(eff)))$$

with $com_{sha.fix}(out(eff,inl,ran)$ as the worker's compensation for a given output, dis(eff) as his disutility for his effort and α as the worker's *absolute risk aversion*. The worker is strictly *risk averse*: $\alpha > 0$.

In addition it is assumed, that dis(eff) is expressed in *monetary* terms with:

(10) $$dis(eff) = d \cdot eff$$

with d as the constant factor for disutility.

Maintaining the inventories, causes inventory costs, which are linear:

(11) $$co_{inv}(inl) = i \cdot inl$$

with i as the constant inventory costs for maintaining a unit of average inventory level.

The interaction between owner and worker within the agency relationship proceeds according to the following temporal structure:

10) Cf. HOLMSTRÖM (1987).

STAGE 1: The owner offers a ("take-or-leave-it") contract by specifying the fee (fix) and the share (sha).

STAGE 2: The worker decides whether to *accept* or *refute* this contract. If he refutes the contract, then the interaction is over and he gets his *reservation utility*[11)], $\underline{u}_A$. If the worker accepts the contract, then:

STAGE 3: The owner chooses an inventory level, $\text{inl} \in \text{INL}$, and the worker chooses his effort level, $\text{eff} \in \text{EFF}$.

STAGE 4: The worker's effort together with the inventory level and the realization of a random term determines the output, $\text{out}(\text{eff}, \text{inl}, \text{ran})$.

STAGE 5: The owner observes the realized output and the worker get the compensation according to the arrangements in the contract.

The only variable which is publicly observed is the realized output. Thus, the contract must take the form of a compensation of the worker that only depends on the output. The owner has to choose the compensation that maximizes his *expected utility* Eu_o:

$$\max_{\text{sha},\text{fix},\text{inl},\text{eff}} \left[Eu_o \left(\text{out}(\text{eff},\text{inl},\text{ran}), \text{com}_{\text{sha.fix}}(\text{out}(\text{eff},\text{inl},\text{ran})), \text{co}_{\text{inv}}(\text{inl}) \right) \right] \tag{12}$$

$$= \max_{\text{sha},\text{fix},\text{inl},\text{eff}} \left[\int_{-\infty}^{+\infty} \left(\text{out}(\text{eff},\text{inl},\text{ran}) - \text{com}_{\text{sha.fix}}(\text{out}(\text{eff},\text{inl},\text{ran})) - \text{co}_{\text{inv}}(\text{inl}) \right) f(\text{ran})\, d\text{ran} \right]$$

$$= \max_{\text{sha},\text{fix},\text{inl},\text{eff}} \left[\theta(\text{eff},\text{inl}) - (\text{sha} \cdot \theta(\text{eff},\text{inl}) - \text{fix}) - i \cdot \text{inl} \right].$$

The owner will take into account the consequences of his contract to the worker's action. The worker will only choose that effort level, which maximizes his *expected utility* Eu_w (*incentive compatibility constraint*):

(13) $$\text{eff}^* \in \underset{\text{eff}}{\arg\max}\left[\text{Eu}_{\text{w}}\left(\text{com}_{\text{sha.fix}}(\text{out}(\text{eff},\text{inl},\text{ran})),\text{dis}(\text{eff})\right)\right]$$

$$= \text{eff}^* \in \underset{\text{eff}}{\arg\max}[\int_{-\infty}^{+\infty}(-\exp(-a(\text{sha}\cdot(\theta(\text{eff},\text{inl})+\text{ran})\text{-fix-dis}(\text{eff})))\ \ f(\text{ran})\ \text{dran}]$$

and he will only accept the contract offer, when he at least gets his reservation utility $\underline{u}_w$ (*participation constraint*):

(14) $$\int_{-\infty}^{+\infty}(-\exp(-a(\text{sha}\cdot(\theta(\text{eff},\text{inl})+\text{ran})\text{-fix-dis}(\text{eff})))\ \ f(\text{ran})\ \ \text{dran} \geq \underline{u}_w\,.$$

In the ADL mini-theory the worker's reservation utility $\underline{u}_w$ is assumed equal zero:

(15) $$\underline{u}_w = 0\,.$$

Therefore, the participation constraint can be modified to:

(16) $$\int_{-\infty}^{+\infty}(-\exp(-a(\text{sha}\cdot(\theta(\text{eff},\text{inl})+\text{ran})\text{-fix-dis}(\text{eff})))\ \ f(\text{ran})\ \ \text{dran} \geq 0\,.$$

The owner's problem is to maximize his expected utility (12) such that the *incentive compatibility constraint* (13) and the *participation constraint* (16) are fulfilled. Because of *normal distributed* output, *linear* contracts and *exponentially* utility function of the worker, the worker's utility function u_w can be expressed as his *certainty equivalent* CE_w:

(17) $$\text{CE}_{\text{w}} = \text{sha}\cdot\theta(\text{eff},\text{inl}) - \text{fix}$$

$$-\,0{,}5\cdot\alpha\cdot\text{Var}(\text{com}_{\text{sha.fix}}(\text{out}(\text{eff},\text{inl},\text{ran}))) - \text{dis}(\text{eff})$$

11) The reservation utility $\underline{u}_w$ is the worker's minimum expected utility attained via a market or negotiation process.

$$= \text{sha} \cdot \theta(\text{eff}, \text{inl}) - \text{fix} - 0{,}5 \cdot \alpha \cdot \text{sha}^2 \cdot \sigma^2_{\text{out}} - \text{d} \cdot \text{eff}$$ [12].

After reformulating the worker's *incentive compatibility constraint* (13) and the *participation constraint* (16) with the certainty equivalent, the owner's maximization problem is:

(18) $$\max_{\text{fix,sha,inl,eff}} \left[\theta(\text{eff}, \text{inl}) - (\text{sha} \cdot \theta(\text{eff}, \text{inl}) - \text{fix}) - \text{i} \cdot \text{inl}\right]$$

subject to:

(19) $$\text{eff}^* \in \arg\max_{\text{eff}} \left[\text{sha} \cdot \theta(\text{eff}, \text{inl}) - \text{fix} - 0{,}5 \cdot \alpha \cdot \text{sha}^2 \cdot \sigma^2_{\text{out}} - \text{d} \cdot \text{eff}\right]$$

(20) $$\text{sha} \cdot \theta(\text{eff}, \text{inl}) - \text{fix} - 0{,}5 \cdot \alpha \cdot \text{sha}^2 \cdot \sigma^2_{\text{out}} - \text{d} \cdot \text{eff} \geq 0 .$$

The participation constraint (20) will be met as equality[13], that is, the worker's expected compensation will satisfy:

(21) $$\text{sha} \cdot \theta(\text{eff}, \text{inl}) - \text{fix} = 0{,}5 \cdot \alpha \cdot \text{sha}^2 \cdot \sigma^2_{\text{out}} + \text{d} \cdot \text{eff} .$$

This expression can be eliminated by substituting it into the objective function of the owner (18). The *first-order condition* of the worker's incentive compatibility constraint on his effort level is set equal zero[14]. Therefore, the constraint can be expressed as:

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12) See the appendix 1 for the determination of the certainty equivalent CE.

13) For every solution whereby the worker's certainty equivalent is greater than his reservation utility, the owner could decrease the fee (fix) and could thereby increase his expected utility. Therefore, the participation constraint will be met as equality.

14) The substitution of the incentive compatibility constraint (21) by the first-order condition (24) is known as the "*first-order approach*". The "first-order approach" is not valid in general, because first-order conditions are only sufficient for describing an optimum of a concave function. Usually, there is no guarantee that at the optimal compensation $\text{com}_{\text{sha,fix}}(\text{out}(\text{eff}, \text{inl}, \text{ran}))$ the worker's expected utility function Eu_{w} is concave in the effort level eff. But there are conditions specifying when the substitution is acceptable. ROGERSON introduced in this context the *convexity of the distribution function condition (CDFC)* and the *monotone likelihood ratio condition (MLRC)* as sufficient (but not necessary) conditions for the validity of the first-order-approach; cf. ROGERSON (1985), pp. 1360.
In the mini-theory the first-order approach can be used, because the condition for a global maximum

(22) $\text{sha} \cdot \theta_{\text{eff}}(\text{eff}, \text{inl}) - d = 0$.

After reformulating the incentive compatibility constraint and participation constraint of the worker, the maximization problem of the owner can be written as:

(23) $$\max_{\text{fix,sha,inl,eff}} \left[\theta(\text{eff}, \text{inl}) - 0{,}5 \cdot \alpha \cdot \text{sha}^2 \cdot \sigma^2_{\text{out}} - d \cdot \text{eff} - i \cdot \text{inl}\right]$$

subject to the first-order-condition of the incentive compatibility constraint (22). The owner's maximization problem can be written in Lagrangian form as:

(24) $$\max\left[L(\text{sha}, \text{inl}, \text{eff}, \lambda)\right] = \max\left[\theta(\text{eff}, \text{inl}) - 0{,}5 \cdot \alpha \cdot \text{sha}^2 \cdot \sigma^2_{\text{out}} - d \cdot \text{eff} - i \cdot \text{inl}\right.$$
$$\left. + \lambda(\text{sha} \cdot \theta_{\text{eff}}(\text{eff}, \text{inl}) - d)\right].$$

The solution of this maximization problem generates the *pareto-efficient* contracts, whereby neither owner nor worker can make it better off without the other being making it worse off. The solution of the owner's problem satisfies the following first order conditions:

(25) $\text{sha} = \dfrac{\lambda \cdot \theta_{\text{eff}}(\text{eff}, \text{inl})}{\alpha \cdot \sigma^2_{\text{out}}}$ [15],

(26) $\theta_{\text{inl}}(\text{eff}, \text{inl}) = i - \lambda \cdot \text{sha} \cdot \theta_{\text{eff,inl}}(\text{eff}, \text{inl})$,

(27) $\theta_{\text{eff}}(\text{eff}, \text{inl}) = d - \lambda \cdot \text{sha} \cdot \theta_{\text{eff.eff}}(\text{eff}, \text{inl})$ and

(28) $d = \text{sha} \cdot \theta_{\text{eff}}(\text{eff}, \text{inl})$.

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is fulfilled: $\text{sha} \cdot \theta_{\text{eff.eff}}(\text{eff}, \text{inl}) \le 0$, due to: $\theta_{\text{eff.eff}}(\text{eff}, \text{inl}) < 0$ (equitation 5) and $\text{sha} \in \Re_{\ge 0}$ (equitation 7).

15) Note, that the mean $\theta(\text{eff}, \text{inl})$ and the variance $\sigma^2_{\text{out}} = \text{Var}(\text{out}(\text{eff}, \text{inl}, \text{ran}))$ of the normally distributed output are independent parameters. Changing the mean doesn't affect the variance and vice versa. Therefore, neither an increase of the effort level nor an increase of the inventory level affects the variance of the output: $\text{Var}_{\text{eff}}(\text{out}(\text{eff}, \text{inl}, \text{ran})) = 0$ and $\text{Var}_{\text{inl}}(\text{out}(\text{eff}, \text{inl}, \text{ran})) = 0$. See appendix 2 for proof.

These results could be used for comparative statics. For instance, it can be illustrated, how the effort level will change, if the output-dependent share will change.

2.2 Structuring Defect

A nomological hypothesis is the central component of a scientific theory. Within the philosophical community there is no clear answer regarding the question about the necessary and sufficient conditions for classifying a statement as a nomological hypothesis[16)]. Therefore, a working definition for a nomological hypothesis is required, which reflects the *intuitive*, *pre-theoretical* understanding of a nomological hypothesis: In this paper a nomological hypothesis will be understood as a (*non-tautological*) *universal quantified subjugat*. *Prima facie*, there are no universal quantified subjugats in the mini-theory, because the mini-theory is formulated according to the conventional conception of theories. Regarding the structure only two components can be identified:

- On the one hand there are *axioms* (equations (1)-(3) and (5)-(15)) and
- on the other hand there a *theorems* (equations (4) and (16)-(28)).

Beside this minimal structure, the mini-theory posses no further structure. In particular, there are no statements, which are explicitly representing nomological hypotheses. Thus, the *nomological essence* of the mini-theory is not articulated clearly. It could be asserted that the obscurity of the nomological essence of the mini-theory is not a problem. This assertion is insofar true, as it is only intended to *describe* certain aspects of reality. But theories generally and the regarded mini-theory in particular are designed not only to describe certain phenomena but also to *explain* these phenomena. As a matter of fact, with the mini-theory it is intended to explain motivational aspects of JIT production systems. An explanation requires for example in the case of the *deductive nomological explanation*[17)] at least one nomological hypothesis. Therefore, the mini-theory has to contain at least one explicitly formulated nomological hypothesis to achieve the aim of explanation. In order to overcome the structuring defect, a theory conception is required which allows the modelling of the essential components of a the-

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16) See for different definitions for nomological hypotheses ALBERT (1998), p. 45 and pp. 80; OPP (2002), pp. 120; RAPPAPORT (1998), p. 55.

ory. Therefore, the mini-theory is reconstructed from the *structuralist point of view*.

3 The Structuralist Conception of Theories

The structuralist conception of theories[18)] means the view of the scientific theories initially presented by Sneed and enhanced and elaborated beside others by Stegmüller and Balzer[19)]. In addition to Sneeds original application to natural science (in particular physics) structuralism has been applied also to the social sciences[20)], i.e. to economics[21)] and production theory[22)]. It is impossible to lay out all aspects of the structuralism in a few pages[23)]. Nevertheless, we want to try to highlight some aspects of structuralism, which are essential for the argument.

According to conventional conception of theories, a scientific theory is a set of systematically connected statements completely closed under deduction[24)]. Only two components were distinguished:

- *axioms*, which are those statements assumed as true without proof, and
- *theorems,* which can be derived directly or indirectly from the axioms.

In contrast to the conventional conception of theories, according to structuralism a scientific theory is a complex, multi-layered mathematical object. On the first layer, a theory T is considered as an ordered pair[25)]

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17) Cf. for the deductive nomological explanation HEMPEL (1977), pp. 8.

18) The structuralist conception of theories is also designated as "*structuralist approach*", "*set-theoretic structuralism*", "*structuralist program*" or "*non-statement-view*". Henceforth it will be denoted shortly as "*structuralism*".

19) Cf. SNEED (1971); STEGMÜLLER (1980); BALZER (1987).

20) See for a survey of the bibliography of structuralism DIEDERICH (1994) and for example the structuralist reconstruction of the system theory according to LUHMANN .

21) Cf. for example BALZER (1982); HAMMINGA (1984); DE LA SIENRA (2000).

22) Cf. ZELEWSKI (1993).

23) For a detailed description of the structuralism cf. BALZER (1987) and BALZER (1996).

24) Cf. for a detailed description of the conventional conception of theories BUNGE (1967), pp. 406.

25) In this paper the following symbols will be used: "$\forall(...):$": "for all…"; "$\rightarrow$": implies, "$\wedge$": logical conjunction, "$\subseteq$": subset of, "$\Re$": real numbers, "$\Re_{\geq 0}$": non-negative real numbers, "$\times$": Carte-

$$T = \langle K, I \rangle .$$

In this ordered pair, K is the *(formal) core* of the theory and I is the *set of intended applications*. The core K itself consists (on the second layer) of four different sets:

$$K = \langle M_p, M, M_{pp}, C \rangle .$$

The core K contains the *set of potential models* M_p, the *set of models* M, the *set of partial potential models* M_{pp} and the *set of constraints* C. *Potential models* for the theory T are entities for which it makes sense to attempt to apply the theory and that have at least enough structural similarity to it. Formally, potential models are set-theoretic entities that can be formulated in the vocabulary of the theory T. Not all potential models of the theory are models of the theory. Only those potential models become a model of the theory, which fulfil the nomological hypotheses of the theory, that is:

$$M \subseteq M_p .$$

A theory can have different (possibly overlapping) applications. Therefore within structuralism the set of *constraints* C is defined. The constraints ensure that the same entity appearing in different applications is used in identical ways in all applications[26)]. The explication of the essentials of structuralism requires the introduction of an additional concept: *T-theoreticity*. Structuralists argue that it is not possible to empirically test a theory as long as there is at least one *T-theoretical* term. A term t is called T-theoretical if all methods to measure its value presuppose that the theory T is true according to at least one intended application. Every attempt to empirically test a theory with at least one T-theoretical term results either in a *circulus vitiosus* or in an *infinite regress*. The structuralist method of getting around the problem of T-theoretical terms is to consider the *set of partial potential models* M_{pp}. The set of partial potential models is derived from the set of potential models by "lopping off" the T-theoretical terms from the description of the elements in M_p applying the so called Ramsey operator "r". The set of

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sian product, "$\in$": element of, "pot_+": power set of a set without the empty set, "$< ... >$": tuple. The other symbols will be defined at the used locations.

26) There are difficulties to identify the constraints of the regarded mini-theory. The only constraint, which could be defined, is the *identity constraint*, which asserts that two entities have identical properties in two different applications of a theory. But this constraint does not hold here, because the utility functions of an owner or worker may easily change from one model to another. Therefore the

partial potential models contains only those entities, which are described in the T-non-theoretical vocabulary of the theory T[27]. The empirical content of the theory remains unchanged because of the special nature of the Ramsey operation.

Hitherto, the core describes the mathematical structure of a theory. But theories are entities, which are constructed to be applied to reality. The *set of intended applications* I is the set (of representations) of substantiate real entities which the theory is about. The set of intended applications is an *open* set which is only *loosely* specified. One restriction to the set of intended applications is that its members must have the same mathematical structure as the members of the partial potential models: the set of intended applications must be a subset of the set of partial potential models:

$$\mathrm{I} \subseteq \mathrm{M}_{pp} \, .$$

The *intended* applications of a theory are described in the T-non-theoretical vocabulary of the theory, i.e. after applying the Ramsey operator r. Furthermore an *application* of the theory is not only described with the full vocabulary of the theory but fulfils the nomological hypotheses *and* the constraints, that is:

$$\mathrm{I} \subseteq \mathrm{r}[\mathrm{pot}_{+}(\mathrm{M}) \cap \mathrm{C}] \, .$$

Table 1 summarizes the characteristic structure of a theory from the structuralist point of view.

class of constraints is equal the class of potential models (or the power set of the potential models).

27) In agency theory generally there are no *agency-theoretical* terms. In the fist place, “*utility*” is a *possible* agency-theoretical term. But the term utility cannot be agency-theoretical, because there is at least one method (e.g.: *conjoint measurement*) outside the agency theory, which allows measuring the utility of an actor. Therefore, the class of potential models and the class of partial potential models are identical.

theory	$T = < K, I >$
core	$K = < M_p, M, M_{pp}, C >$
intended applications	$I \subseteq r[pot_+(M) \cap C]$

Table 1: Structure of a theory from the structuralist point of view

According to structuralism a theory is specified with set-theoretic axioms. In this paper, we depart from the set-theoretic axiomatization and embed the *sorted predicate logic* (SPL) into structuralism for two reasons. First, by the sorted predicate logic it is possible to state very clearly the nomological essence of a theory. Second, it is possible to specify intuitively those classes of objects of a theory by the "*sorts*", which are of interest. Table 2 gives an overview of the scheme for a "*well-defined*" theory according to (SPL-)structuralism.

Theory T
(a) vocabulary (set M_p of potential models)
(aa) sorts (ab) function symbols (ac) predicate symbols (ad) definitional relations
(b) nomological hypotheses (set M of models)
(c) application conditions (set I of intended applications)
(ca) interpretation conditions (ca) domains of the sorts (cb) mapping conditions for the function symbols (cc) extensions of the predicate symbols (cb) boundary conditions

Table 2: Scheme for a theory formulated according to structuralism

4 A Reconstruction of the Mini-Theory

The mini-theory is reconstructed on the basis of the scheme defined above by specifying the vocabulary, nomological hypotheses and the application conditions. Because the aim of this paper is to overcome the structuring defect regarding the nomological essence of the mini-theory, the boundary conditions will not be specified formally.

(a) Vocabulary (set M_p of potential models)

The vocabulary of the mini-theory is specified by introducing the *sorts*, *function symbols*, *predicate symbols* and *definitional relations*. To be able describing the mini-theory, *sorts* are defined for every class of objects (aa). These sorts are those terms, which are of interest within the mini-theory. With these sorts the *function symbols* can be defined (ab). In opposite to functions, a function symbol only specifies the structure of a function and consists of the involved sorts. The *predicate symbols* are introduced to be able formulating the nomological hypothesis of the mini-theory (ac). For the mini-theory *two* predicate symbols are defined: These predicate symbols refers to

- the owner's decision concerning the compensation components (fix, sha) and the inventory level (inl) and
- the worker's decision concerning his effort level (eff).

Finally, the definitional relation specifies the dependencies between the elements of the vocabulary (ad). For the mini-theory only one definitional relation is specified: the cumulative density function of the random term is equal one.

(aa) Sorts

compensation

disutility

effort_level

expected_utility_owner

expected_utility_worker

fixed

inventory_cost

inventory_level

output

probability

random_term

reservation_utility

share

utility_owner

utility_worker

(ab) Function symbols

co_{inv}:	inventory_level $\rightarrow$ inventory_costs
dis:	effort_level $\rightarrow$ disutility
out:	effort_level inventory_level random_term $\rightarrow$ output
com:	output $\rightarrow$ compensation
u_o:	output compensation inventory_cost $\rightarrow$ utility_owner
u_w:	compensation disutility $\rightarrow$ utility_worker
f:	random_term $\rightarrow$ probability
Eu_o:	probability utility_owner $\rightarrow$ expected_utility_owner
Eu_w:	probability utility_worker $\rightarrow$ expected_utility_worker

(ac) Predicate symbols

owner_decision:	fixed share inventory_level
worker_decision:	effort_level

(ad) Definitional relations

$$\forall\left(\text{ran} \in \text{DOM}_{\text{random\_term}}, f(\text{ran}) \in \text{DOM}_{\text{probability}}\right): \int_{-\infty}^{+\infty} f(\text{ran})\,d\text{ran} = 1$$

(b) Nomological Hypotheses (set M of models)

Every phenomenon which can be described in the vocabulary of the mini-theory is a potential model of this mini-theory. To become a model, the phenomenon must also fulfil the nomological hypotheses of the mini-theory. To identify the nomological hypotheses, it is necessary to clarify the sequential decisions by the owner and the worker:

There are decisions to be made by the owner and worker concerning the compensation, inventory and effort level. The owner has to choose the compensation components and inventory level and the worker his effort level. For the given decisions of the owner concerning the compensation components fixed and share as well as the inventory level, the worker chooses a decision eff* that maximizes his *expected utility* Eu_w, if he accepts the contract:

$$\begin{aligned} &Eu_w\left(com_{sha.fix}(out(eff*, inl, ran)), dis(eff*)\right) \\ &= \max\left[\underline{u}_w; Eu_w\left(com_{sha.fix}(out(eff*, inl, ran)), dis(eff*)\right)\right]. \end{aligned}$$

The owner is only interested in the net profits when making his decisions. That is, he maximizes his gross profit minus the compensation for the worker and the costs for maintaining the inventory. Given a decision eff of the worker, the owner chooses the compensation by determining fix* and sha* and the inventory level inl* to maximize his expected utility Eu_o:

$$Eu_o\left(out(eff, inl*, ran), com_{sha*.fix*}(out(eff, inl*, ran)), co_{inv}(inl*)\right).$$

Both (owner and worker) make their decisions in that way, that no one has an incentive to make a decision other than their optimal decision: $sha*, fix*, inl*$ and $eff*$. In this *equilibrium* the decisions are *Pareto-efficient.* In this connection the link between agency theory and game theory becomes clear. In the vocabulary of game theory the decisions of owner and worker are their *strategies*. Owner and worker will choose their strategies in that way, that the observed strategies constitute a *Nash-equilibrium*[28].

Now, the only one nomological hypothesis NH of the mini-theory can be specified: When the decisions of owner (fix*,sha*,inl*) and worker (eff*) are observed, then owner and worker have made their decisions that form an *equilibrium*:

28) Games played by owner and worker can lead to many Nash-equilibria and agency-theory claims that every equilibrium can be achieved; cf. SCHWEIZER (1999), pp. 24.

$$NH :\Leftrightarrow \forall(sha^* \in DOM_{share}, fix^* \in DOM_{fixed}, inl^* \in DOM_{inventory\_level}, \ldots$$

$$eff^* \in DOM_{effort\_level}, \underline{u}_w \in DOM_{reservation\_utility}):$$

$$owner\_decision(sha^*, fix^*, inl^*) \wedge worker\_decision(eff^*) \rightarrow \ldots$$

$$\big(eff^* = \arg\max\left[\underline{u}_w ; Eu_w \left(com_{sha.fix}(out(eff^*, inl, ran)), dis(eff^*)\right)\right] \wedge \ldots$$

$$(sha^*, fix^*, inl^*) = \arg\max\left[Eu_o(out(eff, inl^*, ran), com_{sha^*.fix^*}(out(eff, inl^*, ran)), co_{inv}(inl^*))\right]\big)$$

(c) Application conditions

The application conditions consist of the interpretation conditions and the boundary conditions. The interpretation conditions concrete the abstract sorts, function and predicate symbols, which are defined in the section vocabulary. Every sort is interpreted through a domain (DOM) of admissible terms (constants or variables). For the function symbols those mapping conditions were used, which are formulated in the mini-theory. It is not necessary to define the extensions of the predicate symbols of the mini-theory, because the extensions of the predicate symbols are determined theory-endogenously by the "arg max"-operators. The boundary conditions isolate the intended applications of the mini-theory on those aspects of reality, which are intended to explain with it. The boundary conditions will be left out in this paper[29)].

(ca) Interpretation conditions

(caa) Domains of the sorts (DOM)

$$DOM_{compensation} = \Re_{\geq 0}$$

$$DOM_{disutility} = \Re_{\geq 0}$$

$$DOM_{effort\_level} = \Re_{\geq 0}$$

29) One boundary condition is the worker's zero reservation utility. The mini-theory only intends to explain those owner-worker relationships, where the worker's reservation utility is equal zero: $\underline{u}_w=0$.

$\mathrm{DOM}_{\mathrm{expected\_utility\_owner}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{expected\_utility\_worker}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{fixed}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{inventory\_costs}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{inventory\_level}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{output}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{probability}} = [0;1]$

$\mathrm{DOM}_{\mathrm{random\_term}} = \Re$

$\mathrm{DOM}_{\mathrm{reservartion\_utility}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{share}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{utility\_owner}} = \Re_{\geq 0}$

$\mathrm{DOM}_{\mathrm{utility\_worker}} = \Re_{\geq 0}$

(cab) Mapping conditions for the functions

$\mathrm{co}_{\mathrm{inv}}$: $\mathrm{DOM}_{\mathrm{inventory\_level}} \to \mathrm{DOM}_{\mathrm{inventory\_costs}}$

$(\mathrm{inl}) \to \mathrm{co}_{\mathrm{inv}}(\mathrm{inl}) = \mathrm{i} \cdot \mathrm{inl}$

dis: $\mathrm{DOM}_{\mathrm{effort\_level}} \to \mathrm{DOM}_{\mathrm{disutility}}$

$(\mathrm{eff}) \to \mathrm{dis}(\mathrm{eff}) = \mathrm{d} \cdot \mathrm{eff}$

out: $\mathrm{DOM}_{\mathrm{effort\_level}} \times \mathrm{DOM}_{\mathrm{inventory\_level}} \times \mathrm{DOM}_{\mathrm{random\_term}} \to \mathrm{DOM}_{\mathrm{output}}$

$(\mathrm{eff}, \mathrm{inl}, \mathrm{ran}) \to \mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran}) = \theta(\mathrm{eff}, \mathrm{inl}) + \mathrm{ran}$

com: $\mathrm{DOM}_{\mathrm{output}} \to \mathrm{DOM}_{\mathrm{compensation}}$

$\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran}) \to \mathrm{com}_{\mathrm{sha.fix}}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran})) = \mathrm{sha} \cdot (\theta(\mathrm{eff}, \mathrm{inl}) + \mathrm{ran}) - \mathrm{fix}$

u_{o}: $\mathrm{DOM}_{\mathrm{output}} \times \mathrm{DOM}_{\mathrm{compensation}} \times \mathrm{DOM}_{\mathrm{inventory\_cost}} \to \mathrm{DOM}_{\mathrm{utility\_owner}}$

$(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran}), \mathrm{com}_{\mathrm{sha.fix}}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran})), \mathrm{co}_{\mathrm{inv}}(\mathrm{inl})) \to$

$$u_o(\mathrm{out}(\mathrm{eff},\mathrm{inl},\mathrm{ran}),\mathrm{com}_{\mathrm{sha.fix}}\mathrm{out}(\mathrm{eff},\mathrm{inl},\mathrm{ran}),\mathrm{co}_{\mathrm{inv}}(\mathrm{inl}))$$

$$= (\theta(\mathrm{eff},\mathrm{inl}) + \mathrm{ran}) - (\mathrm{sha}\cdot(\theta(\mathrm{eff},\mathrm{inl}) + \mathrm{ran}) - \mathrm{fix}) - i\cdot\mathrm{inl}$$

u_W: $\mathrm{DOM}_{\mathrm{compensation}} \times \mathrm{DOM}_{\mathrm{disutility}} \rightarrow \mathrm{DOM}_{\mathrm{utility\_worker}}$

$$(\mathrm{com}_{\mathrm{sha.fix}}(\mathrm{out}(\mathrm{eff},\mathrm{inl},\mathrm{ran})),\mathrm{dis}(\mathrm{eff})) \rightarrow u_w(\mathrm{com}_{\mathrm{sha.fix}}(\mathrm{out}(\mathrm{eff},\mathrm{inl},\mathrm{ran})),\mathrm{dis}(\mathrm{eff}))$$

$$= -\exp(-a(\mathrm{sha}\cdot(\theta(\mathrm{eff,inl}) + \mathrm{ran})\text{-fix-}d\cdot\mathrm{eff}))$$

f: $\mathrm{DOM}_{\mathrm{random\_term}} \rightarrow \mathrm{DOM}_{\mathrm{probability}}$

$$\mathrm{ran} \rightarrow f(\mathrm{ran}) = \frac{1}{\sqrt{2\pi \mathrm{Var}(\mathrm{ran})}} \exp\left(-\frac{1}{2}\left(\frac{(\mathrm{ran} - E(\mathrm{ran}))^2}{\mathrm{Var}(\mathrm{ran})}\right)\right)$$

Eu_o: $\mathrm{DOM}_{\mathrm{probability}} \times \mathrm{DOM}_{\mathrm{utility\_owner}} \rightarrow \mathrm{DOM}_{\mathrm{expected\_utility\_owner}}$

$$(f(\mathrm{ran}), u_o) \rightarrow Eu_o(f(\mathrm{ran}), u_o)$$

$$= \int_{-\infty}^{+\infty} \left((\theta(\mathrm{eff},\mathrm{inl}) + \mathrm{ran}) - (\mathrm{sha}\cdot(\theta(\mathrm{eff},\mathrm{inl}) + \mathrm{ran}) - \mathrm{fix}) - i\cdot\mathrm{inl}\right) f(\mathrm{ran})\, d\mathrm{ran}$$

Eu_W: $\mathrm{DOM}_{\mathrm{probability}} \times \mathrm{DOM}_{\mathrm{utility\_worker}} \rightarrow \mathrm{DOM}_{\mathrm{expected\_utility\_worker}}$

$$(f(\mathrm{ran}), u_w) \rightarrow Eu_w(f(\mathrm{ran}), u_w)$$

$$\int_{-\infty}^{+\infty} (-\exp(-a(\mathrm{sha}\cdot(\theta(\mathrm{eff,inl}) + \mathrm{ran})\text{-fix - dis(eff)}))\;\; f(\mathrm{ran})\;\; d\mathrm{ran}$$

5 Concluding remarks

The aim of this paper was to show that the conventional formulated ADL-model is characterized by a structuring defect. By the structuralist reconstruction this defect is being overcome and the nomological essence of the mini-theory can be identified: If decisions of owner and worker concerning compensation components, inventory level and effort level are observed, than these decisions are forms of rational strategic behaviour and constitute an equilibrium. But this "*agency*" equilibrium demands too much of human agents. No human agent (neither owner nor worker) will be able to choose his strategy in that way, that the decisions will meet in equilibrium. Therefore, the nomological hypothesis should be understood rather in the sense of a *tendency* of human agents toward equilibrium. It is considerable, that despite the criticism of the model of rational actor since 50 years, agency theory has not been rejected. In opposite it can be determined, that *new* models of agency theory are developed (so the regarded mini-theory), which are intended to explain certain phenomena. It is remarkable, that the agency theory is obviously characterized by immunity against empirical refutations. Beside the problem regarding the empirical adequacy of the mini-theory, another topic has to be emphasized: the analysis of the dependencies of the mini-theory to other production management theories. By the reconstruction of the mini-theory it has now a specific structure, which allows the analysis of the *inter-theoretical links* between the mini-theory and other theories. Thus, for instance it can be analyzed, whether there is a relation to activity theory or other theories from production management as well from other economic domains. In particular, it is a task for further research to include more aspects of JIT production systems into the formal theory representation. There is a gap between the intention to explain certain aspects of reality and it's formally representation. Although with the mini-theory it is claimed to explain motivational aspects of *JIT production systems*, there are no JIT-specific components in the formal representation. This gap will be closed in the future by an extension of the structuralist reconstruction presented in this paper.

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Appendix 1

The appendix describes how the certainty equivalent CE is determined. Because of *normal distributed* output, *linear* contracts (c) and *exponentially* utility function the maximization of the expected utility is equal to the maximization of the certainty equivalent CE, that is:

(A1.1) u(CE)=Eu(c) $\Leftrightarrow -\exp(-\alpha CE) = E(-\exp(-\alpha c))$

Note, that μ is the expected compensation ($sha \cdot \theta(eff, inl) - fix$) and σ^2 is the variance of the compensation ($Var(com_{sha.fix}(out(eff, inl, ran)))$).

Let's modify the second term of (A1.1):

(A1.2) $E(-\exp(-\alpha c)) = \int -\exp(-\alpha c) f(c)\, dc$

$$= \int -\exp(-\alpha c) \frac{1}{\sqrt{2\pi\sigma^2}} \exp(0,5\,(c-\mu)^2 / \sigma^2)\, dc$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int -\exp(-\alpha c - (0,5\,(c-\mu)^2 / \sigma^2)\, dc$$

Auxiliary calculation:

$$(-\alpha c) - (0,5\,(c-\mu)^2 / \sigma^2)$$

$$= -0,5\,((c^2 - 2c\mu + \mu^2 + 2\alpha c\sigma^2) / \sigma^2) = -0,5\,((c^2 - 2c\mu + \mu^2 + 2\alpha c\sigma^2) / \sigma^2)$$

$$= -0,5\,((c^2 - 2c(\mu - \alpha\sigma^2) + (\mu - \alpha\sigma^2)^2 + 2\mu\alpha\sigma^2 - \alpha^2\sigma^4) / \sigma^2)$$

$$= -0,5\,((c^2 - (\mu - \alpha\sigma^2))^2 + \alpha\sigma^2(2\mu - \alpha\sigma^2)) / \sigma^2)$$

$$= -0,5 \frac{(c^2 - (\mu - \alpha\sigma^2))^2}{\sigma^2} - 0,5 \frac{\alpha\sigma^2(2\mu - \alpha\sigma^2)}{\sigma^2}$$

$$= -0,5 \frac{(c^2 - (\mu - \alpha\sigma^2))^2}{\sigma^2} - 0,5\,\alpha(2\mu - \alpha\sigma^2)$$

$$= -0,5\frac{(c^2-(\mu-\alpha\sigma^2))^2}{\sigma^2} - \alpha\mu + 0,5\alpha^2\sigma^2$$

End of auxiliary calculation

Substituting

$$(-\alpha c) - (0,5\,(c-\mu)^2 / \sigma^2)$$

by

$$-\,0,5\frac{(c^2-(\mu-\alpha\sigma^2))^2}{\sigma^2} - \alpha\mu + 0,5\alpha^2\sigma^2$$

leads to:

(A1.3) $$E(-\exp(-\alpha c)) = \frac{1}{\sqrt{2\pi\sigma^2}}\int -\exp(-0,5\frac{(c^2-(\mu-\alpha\sigma^2))^2}{\sigma^2} - \alpha\mu + 0,5\alpha^2\sigma^2)\;dc$$

$$= \int -\exp(-\alpha\mu + 0,5\alpha^2\sigma^2)\,\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-0,5\frac{(c^2-(\mu-\alpha\sigma^2))^2}{\sigma^2})\;dc$$

$$= -\exp(-a\mu + 0,5a^2\sigma^2)\underbrace{\int \frac{1}{\sqrt{2\pi\sigma^2}}\exp(-0,5\frac{(c^2-(\mu-a\sigma^2))^2}{\sigma^2})\;dc}_{\substack{\text{This is the density function for a normal distribution with mean} \\ (\mu-a\sigma^2)\text{ und variance }\sigma^2\text{.By definition that is:} \\ \int -\exp(c/(\mu-a\sigma^2);\sigma^2)\,dc\ =1}}$$

Therefore, this expression can be written as:

(A1.4) $$E(-\exp(-\alpha c)) = -\exp(-\alpha\mu + 0,5\alpha^2\sigma^2)$$

After substitution of this term into (A1.1):

(A1.5) $$-\exp(-\alpha CE) = -\exp(-\alpha\mu + 0,5\alpha^2\sigma^2)$$

After eliminating the exponential function and multiplication with $-1/\alpha$ the equation can be written as:

(A1.6) $$CE = \mu - 0,5\alpha\sigma^2.$$

The terms μ and σ^2 are the expected compensation of the worker ($sha \cdot \theta(eff, inl) - fix$) and the variance of the compensation ($Var(com_{sha.fix}(out(eff, inl, ran)))$), respectively. Therefore, (A1.6) can be written as:

(A1.7) $CE = sha \cdot \theta(eff, inl) - fix - 0,5 \cdot \alpha \cdot Var(com_{sha.fix}(out(eff, inl, ran)))$.

The variance of the compensation is defined as:

(A1.8) $Var(com_{sha.fix}(out(eff, inl, ran)))$

$$= \int [com_{sha.fix}(out(eff, inl, ran)) - E[com_{sha.fix}(out(eff, inl, ran))]]^2 \cdot f(out)\, dout \,.$$

After applying the calculation rule for linear transformed variances[30), the variance of the compensation (A1.8) can be expressed as:

(A1.9) $Var(com_{sha.fix}(out(eff, inl, ran))) = sha^2 \cdot Var(out(eff, inl, ran))) = sha^2 \cdot \sigma^2_{out}$.

Thus, (A1.7) can be written as:

(A1.10) $CE = sha \cdot \theta(eff, inl) - fix - 0,5 \cdot \alpha \cdot sha^2 \cdot \sigma^2_{out}$.

---

30) The variance of a linear transformed random term x with $g(x) = b \cdot x + a$ and variance $Var(x)$ is:

$$Var(g(x)) = \int ((b \cdot x + a) - E(b \cdot x + a))^2 = b^2 \cdot Var(x) \,.$$

Appendix 2

The mean and the variance of the normally distributed output are independent parameters. Changing the mean doesn't affect the variance and vice versa. The increase of the effort level or the inventory level affects the mean of the output $\theta(\mathrm{eff}, \mathrm{inl})$, but not the variance of the output:

$$\mathrm{Var}_{\mathrm{eff}}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran})) = 0 \text{ and } \mathrm{Var}_{\mathrm{inl}}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran})) = 0 .$$

In order to proof this proposition, it must be shown, that the variance of the output is independent from the effort level as well as from the inventory level. The variance of the output is defined as:

$$\text{(A2.1)}\quad \mathrm{Var}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran}))$$

$$= \int [\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran}) - \mathrm{E}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran})))^2 \cdot \mathrm{f}(\mathrm{out})\, \mathrm{dout} .$$

After applying the calculation rule for linear transformed variances, the variance of the output can be expressed as:

$$\text{(A2.2)}\quad \mathrm{Var}(\mathrm{out}(\mathrm{eff}, \mathrm{inl}, \mathrm{ran})) = \mathrm{Var}(\mathrm{ran}) = \sigma^2_{\mathrm{ran}} .$$

That is, the variance of the output is equal to the variance of the random term. By this expression it becomes clear: Neither effort level nor inventory level are parameters of the variance of the random term σ^2_{ran}. Consequently the variance of the output is independent of the effort level and inventory level.